

the more expensive mirror reflex cameras allow for superb quality photographic images. The optics of these camera lenses is beyond the scope of this textbook.

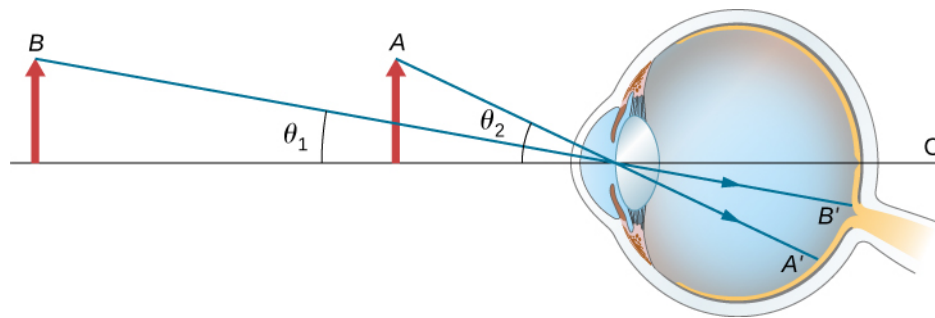
## 2.7 | The Simple Magnifier

### Learning Objectives

By the end of this section, you will be able to:

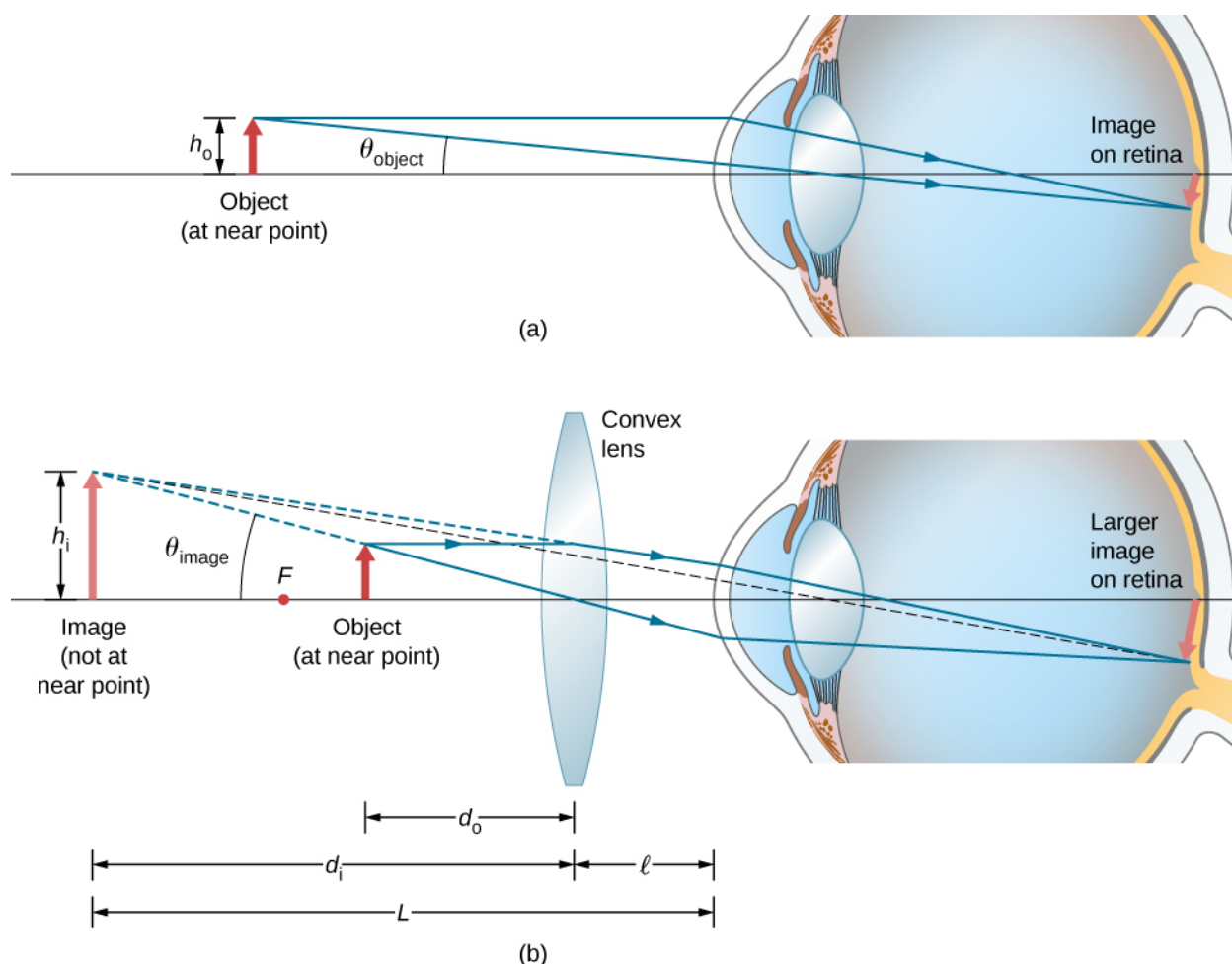
- Understand the optics of a simple magnifier
- Characterize the image created by a simple magnifier

The apparent size of an object perceived by the eye depends on the angle the object subtends from the eye. As shown in **Figure 2.36**, the object at *A* subtends a larger angle from the eye than when it is positioned at point *B*. Thus, the object at *A* forms a larger image on the retina (see  $OA'$ ) than when it is positioned at *B* (see  $OB'$ ). Thus, objects that subtend large angles from the eye appear larger because they form larger images on the retina.



**Figure 2.36** Size perceived by an eye is determined by the angle subtended by the object. An image formed on the retina by an object at *A* is larger than an image formed on the retina by the same object positioned at *B* (compared image heights  $OA'$  to  $OB'$ ).

We have seen that, when an object is placed within a focal length of a convex lens, its image is virtual, upright, and larger than the object (see part (b) of **Figure 2.26**). Thus, when such an image produced by a convex lens serves as the object for the eye, as shown in **Figure 2.37**, the image on the retina is enlarged, because the image produced by the lens subtends a larger angle in the eye than does the object. A convex lens used for this purpose is called a **magnifying glass** or a **simple magnifier**.



**Figure 2.37** The simple magnifier is a convex lens used to produce an enlarged image of an object on the retina. (a) With no convex lens, the object subtends an angle  $\theta_{\text{object}}$  from the eye. (b) With the convex lens in place, the image produced by the convex lens subtends an angle  $\theta_{\text{image}}$  from the eye, with  $\theta_{\text{image}} > \theta_{\text{object}}$ . Thus, the image on the retina is larger with the convex lens in place.

To account for the magnification of a magnifying lens, we compare the angle subtended by the image (created by the lens) with the angle subtended by the object (viewed with no lens), as shown in **Figure 2.37**. We assume that the object is situated at the near point of the eye, because this is the object distance at which the unaided eye can form the largest image on the retina. We will compare the magnified images created by a lens with this maximum image size for the unaided eye. The magnification of an image when observed by the eye is the **angular magnification**  $M$ , which is defined by the ratio of the angle  $\theta_{\text{image}}$  subtended by the image to the angle  $\theta_{\text{object}}$  subtended by the object:

$$M = \frac{\theta_{\text{image}}}{\theta_{\text{object}}} \quad (2.26)$$

Consider the situation shown in **Figure 2.37**. The magnifying lens is held a distance  $\ell$  from the eye, and the image produced by the magnifier forms a distance  $L$  from the eye. We want to calculate the angular magnification for any arbitrary  $L$  and  $\ell$ . In the small-angle approximation, the angular size  $\theta_{\text{image}}$  of the image is  $h_i/L$ . The angular size  $\theta_{\text{object}}$  of the object at the near point is  $\theta_{\text{object}} = h_o/25 \text{ cm}$ . The angular magnification is then

$$M = \frac{\theta_{\text{image}}}{\theta_{\text{object}}} = \frac{h_i (25 \text{ cm})}{L h_o} \quad (2.27)$$

Using **Equation 2.8** for linear magnification

$$m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

and the thin-lens equation

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

in **Equation 2.27**, we arrive at the following expression for the angular magnification of a magnifying lens:

$$\begin{aligned} M &= \left(-\frac{d_i}{d_o}\right) \left(\frac{25 \text{ cm}}{L}\right) \\ &= -d_i \left(\frac{1}{f} - \frac{1}{d_i}\right) \left(\frac{25 \text{ cm}}{L}\right) \\ &= \left(1 - \frac{d_i}{f}\right) \left(\frac{25 \text{ cm}}{L}\right) \end{aligned} \quad (2.28)$$

From part (b) of the figure, we see that the absolute value of the image distance is  $|d_i| = L - \ell$ . Note that  $d_i < 0$  because the image is virtual, so we can dispense with the absolute value by explicitly inserting the minus sign:  $-d_i = L - \ell$ . Inserting this into **Equation 2.28** gives us the final equation for the angular magnification of a magnifying lens:

$$M = \left(\frac{25 \text{ cm}}{L}\right) \left(1 + \frac{L - \ell}{f}\right) \quad (2.29)$$

Note that all the quantities in this equation have to be expressed in centimeters. Often, we want the image to be at the near-point distance ( $L = 25 \text{ cm}$ ) to get maximum magnification, and we hold the magnifying lens close to the eye ( $\ell = 0$ ). In this case, **Equation 2.29** gives

$$M = 1 + \frac{25 \text{ cm}}{f} \quad (2.30)$$

which shows that the greatest magnification occurs for the lens with the shortest focal length. In addition, when the image is at the near-point distance and the lens is held close to the eye ( $\ell = 0$ ), then  $L = d_i = 25 \text{ cm}$  and **Equation 2.27** becomes

$$M = \frac{h_i}{h_o} = m \quad (2.31)$$

where  $m$  is the linear magnification (**Equation 2.32**) derived for spherical mirrors and thin lenses. Another useful situation is when the image is at infinity ( $L = \infty$ ). **Equation 2.29** then takes the form

$$M(L = \infty) = \frac{25 \text{ cm}}{f} \quad (2.32)$$

The resulting magnification is simply the ratio of the near-point distance to the focal length of the magnifying lens, so a lens with a shorter focal length gives a stronger magnification. Although this magnification is smaller by 1 than the magnification obtained with the image at the near point, it provides for the most comfortable viewing conditions, because the eye is relaxed when viewing a distant object.

By comparing **Equation 2.29** with **Equation 2.32**, we see that the range of angular magnification of a given converging lens is

$$\frac{25 \text{ cm}}{f} \leq M \leq 1 + \frac{25 \text{ cm}}{f}. \quad (2.33)$$

### Example 2.10

#### Magnifying a Diamond

A jeweler wishes to inspect a 3.0-mm-diameter diamond with a magnifier. The diamond is held at the jeweler's near point (25 cm), and the jeweler holds the magnifying lens close to his eye.

- What should the focal length of the magnifying lens be to see a 15-mm-diameter image of the diamond?
- What should the focal length of the magnifying lens be to obtain  $10\times$  magnification?

#### Strategy

We need to determine the requisite magnification of the magnifier. Because the jeweler holds the magnifying lens close to his eye, we can use **Equation 2.30** to find the focal length of the magnifying lens.

#### Solution

- The required linear magnification is the ratio of the desired image diameter to the diamond's actual diameter (**Equation 2.32**). Because the jeweler holds the magnifying lens close to his eye and the image forms at his near point, the linear magnification is the same as the angular magnification, so

$$M = m = \frac{h_i}{h_o} = \frac{15 \text{ mm}}{3.0 \text{ mm}} = 5.0.$$

The focal length  $f$  of the magnifying lens may be calculated by solving **Equation 2.30** for  $f$ , which gives

$$M = 1 + \frac{25 \text{ cm}}{f}$$

$$f = \frac{25 \text{ cm}}{M - 1} = \frac{25 \text{ cm}}{5.0 - 1} = 6.3 \text{ cm}$$

- To get an image magnified by a factor of ten, we again solve **Equation 2.30** for  $f$ , but this time we use  $M = 10$ . The result is

$$f = \frac{25 \text{ cm}}{M - 1} = \frac{25 \text{ cm}}{10 - 1} = 2.8 \text{ cm}.$$

#### Significance

Note that a greater magnification is achieved by using a lens with a smaller focal length. We thus need to use a lens with radii of curvature that are less than a few centimeters and hold it very close to our eye. This is not very convenient. A compound microscope, explored in the following section, can overcome this drawback.

## 2.8 | Microscopes and Telescopes

### Learning Objectives

By the end of this section, you will be able to:

- Explain the physics behind the operation of microscopes and telescopes
- Describe the image created by these instruments and calculate their magnifications

Microscopes and telescopes are major instruments that have contributed hugely to our current understanding of the micro- and macroscopic worlds. The invention of these devices led to numerous discoveries in disciplines such as physics, astronomy, and biology, to name a few. In this section, we explain the basic physics that make these instruments work.